NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE 4295

REFLECTION AND TRANSMISSION OF SOUND BY A SLOTTED WALL SEPARATING TWO MOVING FLUID STREAMS

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SUMMARY

The reflection and transmission coefficients have been determined for a plane sound wave incident on a slotted wall separating two moving fluid streams. This acoustics problem is related to the aerodynamic problem of determining the tunnel-wall interference on an oscillating airfoil in a slotted-throat wind tunnel in that the same boundary condition is involved with one of the two streams at the boundary having zero velocity. In the analysis the wall with discrete slots is replaced by an equivalent homogeneous boundary.

INTRODUCTION

The tunnel-wall interference in a closed wind tunnel containing an oscillating airfoil in two-dimensional, subsonic, compressible flow was discussed in reference 1. Strong pressure peaks were shown to occur in the tunnel near critical combinations of wing frequency, tunnel height, and Mach number. The effect of this resonance condition on the aerodynamic force and moment measurements is such that the wind-tunnel data are rendered inapplicable to airfoils in free air.

In view of the fact that much flutter testing is conducted in slotted- or perforated-throat wind tunnels and in small open jets, extending the work of reference 1 to such cases is important. The problen now becomes much more difficult, however, since acoustic energy is transmitted through the boundary in the form of waves set up in the outside air by the unsteady perturbation of the boundary. Several papers (for example, refs. 1, 2, and 3) appear to neglect the possibility of the transmission of sound waves through the boundary.

Unsteady-state boundary conditions which appear to be correct for the case of the free boundary are discussed in references 4 and 5. The purpose of the present analysis, which is a generalization of that of reference 4, is to discuss an approximate treatment of the unsteady-state boundary conditions at a slotted-wall boundary. The approximation

should be reasonably accurate, provided the wave lengths are long relative to the slot geometry and to the boundary-layer thickness, and provided the computation is concerned with conditions at a location far enough from the wall that the separate effects of individual slots can be neglected and the effect of the boundary can be considered approximately "homogeneous." Since an exact mathematical treatment of the actual physical phenomena occurring at this kind of boundary appears to be prohibitively difficult, the present treatment should be useful in obtaining approximate solutions of unsteady-flow problems involving a slotted-wall boundary. The simplest problem of this kind, that is, the reflection and transmission of a plane sound wave, is discussed in this paper. In addition, there is some discussion of the possibility of applying the boundary conditions to the problem of tunnel-wall interference in a two-dimensional wind tunnel containing an oscillating airfoil.

SYMBOLS

c velocity of sound

d distance between slots

k wave number, $\frac{2\pi}{\lambda}$

k_R wave number of ripple moving along boundary

$l = \frac{d}{\pi} \log_e \csc \frac{\pi}{2} r_o$

p perturbation pressure

R reflection coefficient

r_o ratio of width of slots to distance between slots (open ratio of wall)

T transmission coefficient

t time

U phase velocity at boundary

V free-stream velocity

x,y Cartesian coordinates fixed in space

α angle of incidence

β angle of refraction

$\Gamma = \frac{\rho_2 c_2^2}{\rho_1 c_1^2}$

λ wave length

ρ density

φ velocity potential

$\varphi_0 = \exp\left[ik_R(x - Ut)\right]$

Subscripts:

i incident

r reflected

t transmitted

1 conditions below boundary

2 conditions above boundary

ANALYSIS

Consider an infinite, uniformly slotted wall separating two fluid streams moving at different velocities in a direction parallel to the slots. (See figs. 1 and 2.) The perturbation velocity potential of a plane sound wave of unit amplitude incident from below on the boundary at the angle α is

$$\varphi_i = \exp\left\{ik_1\left[y \cos \alpha + (x - V_1 t)\sin \alpha - ct\right]\right\} \tag{1}$$

The perturbation velocity potentials for the reflected and transmitted waves, respectively, have the forms

$$\varphi_r = R \exp\left\{ik_1\left[-y \cos \alpha + (x - V_1 t)\sin \alpha - ct\right]\right\} \tag{2}$$

$$\varphi_t = T \exp\left\{ik_2\left[y \cos \beta + (x - V_2 t)\sin \beta - ct\right]\right\} \tag{3}$$

where β is the angle of refraction and the amplitudes R and T are, in general, complex. The following analysis concerns the problem of determining R and T:

Two relationships which follow from Rayleigh's discussion of the refraction of plane sound waves (ref. 6) and which are applicable to the slotted boundary as well as to the free boundary are

$$c \csc \alpha + V_1 = c \csc \beta + V_2 = U \tag{4}$$

$$k_1 \sin \alpha = k_2 \sin \beta = k_R \tag{5}$$

where U and k_R are, respectively, the phase velocity and wave number of the ripple moving along the boundary.

Reference 7 shows that, when the waves are long compared with the distance between slots, the streamlines of an unsteady flow normal to a slotted wall resemble those of a steady flow. (See fig. 2.) It may be expected that when there is a relative motion parallel to the slots, the normal component of the "perturbation" flow will resemble that of figure 2.

As shown in reference 8, for example, such a slotted boundary may, for the purpose of estimating the boundary effects at a distance, be effectively replaced by a homogeneous boundary having a certain virtual mass per unit area. This virtual mass represents the change in kinetic energy occurring with the isentropic flow through the slots. The virtual mass is denoted by ρl per unit area for each side of the wall where

$$l = \frac{d}{\pi} \log_e \csc \frac{\pi}{2} r_o \tag{6}$$

The difference in pressure per unit area across the wall can be shown to be the product of the virtual mass of the unit area and the normal acceleration. This is expressed by the equation

$$p_t - \left(p_i + p_r\right) = \rho l \frac{\partial}{\partial y}\left(-\frac{1}{\rho} p_t\right) + \rho l \frac{\partial}{\partial y}\left[-\frac{1}{\rho}\left(p_i + p_r\right)\right] \tag{7}$$

The incident perturbation pressure is given by

$$p_i = -\rho_i\left(\frac{\partial \varphi_i}{\partial t}\right)_{y,x-V_1 t} \tag{8}$$

where the subscripts y and $x - V_1 t$ denote that in taking the partial derivative these variables are held constant. This is equivalent to taking the substantial derivative with respect to t; that is, equation (8) gives the pressure on a particle fixed with respect to a coordinate system moving with stream velocity. Similar expressions hold for p_r and p_t.

A general boundary condition that is used when the boundary is displaced by the perturbation flow is obtained by equating the slopes of the streamlines on the two sides of the boundary. This condition should remain valid when the boundary is slotted, insofar as the boundary can be replaced by a homogeneous sheet having the mass per unit area obtained from equation (6). With respect to a frame of reference moving along the boundary with velocity U, the boundary is at rest. The stream velocity in the lower medium is $-(U - V_1)$ with respect to this system, and in the upper medium it is $-(U - V_2)$. The condition that the slopes of the streamlines are equal on the two sides of the boundary then yields the equation

$$(U - V_1)^{-1} \frac{\partial(\varphi_i + \varphi_r)}{\partial y} = (U - V_2)^{-1} \frac{\partial \varphi_t}{\partial y} \tag{9}$$

The reflection and transmission coefficients R and T, respectively, are found by substituting the expressions from equations (1), (2), and (3) into boundary-condition equations (7) and (9) and solving them simultaneously. The mathematical details, which are elementary but tedious, are summarized in the appendix. The results are

$$R = \frac{\sin 2\alpha + ilk_R(\sin 2\beta \cot \alpha + \sin 2\alpha \cot \beta) - \sin 2\beta}{\sin 2\alpha + ilk_R(\sin 2\beta \cot \alpha + \sin 2\alpha \cot \beta) + \sin 2\beta} \tag{10}$$

and

$$T = \frac{4 \cos \alpha \sin \beta}{\sin 2\alpha + il k_R(\sin 2\beta \cot \alpha + \sin 2\alpha \cot \beta) + \sin 2\beta} \tag{11}$$

The reflection and transmission coefficients for the perturbation pressure p are, respectively,

$$R_p = R \tag{12}$$

and

$$T_p = \frac{k_2}{k_1} T = \frac{2 \sin 2\alpha}{\sin 2\alpha + il k_R(\sin 2\beta \cot \alpha + \sin 2\alpha \cot \beta) + \sin 2\beta} \tag{13}$$

No actual mathematical difficulties are introduced by generalizing the analysis to the case where ρ and c are different on the two sides of the boundary, as was done in reference 4. Algebraic details have been omitted, but the results in this case are

$$R_\Gamma = \frac{\Gamma \sin 2\alpha + il k_R(\sin 2\beta \cot \alpha + \Gamma \sin 2\alpha \cot \beta) - \sin 2\beta}{\Gamma \sin 2\alpha + il k_R(\sin 2\beta \cot \alpha + \Gamma \sin 2\alpha \cot \beta) + \sin 2\beta} \tag{14}$$

and

$$T_\Gamma = \frac{4 \frac{c_2}{c_1} \cos \alpha \sin \beta}{\Gamma \sin 2\alpha + il k_R(\sin 2\beta \cot \alpha + \Gamma \sin 2\alpha \cot \beta) + \sin 2\beta} \tag{15}$$

where

$$\Gamma = \frac{\rho_2 {c_2}^2}{\rho_1 {c_1}^2}$$

COMPARISON WITH PREVIOUS RESULTS

The velocity potential undergoes a change of phase at the boundary as attested by the fact that R and T are complex. This effect is consistent with the known "inductance" effect of a slotted wall (fig. 8 of ref. 9). The fact that this change of phase is due entirely to the

virtual mass at the boundary can readily be demonstrated by setting $l = 0$ in equations (10), (11), and (13). This gives the real quantities

$$R_{l=0} = \frac{\sin 2\alpha - \sin 2\beta}{\sin 2\alpha + \sin 2\beta}$$

$$T_{l=0} = \frac{4 \cos \alpha \sin \beta}{\sin 2\alpha + \sin 2\beta}$$

$$T_{p,l=0} = \frac{2 \sin 2\alpha}{\sin 2\alpha + \sin 2\beta}$$

which are the results obtained in references 4 and 5 for the free-boundary problem. When l approaches ∞ (the condition for a closed tunnel), R approaches 1 and T approaches 0.

When there is no relative motion at the boundary, the quantities $\sin 2\alpha$ and $\sin 2\beta$ are equal. In this case, equations (10) and (11) become, respectively,

$$R_{\alpha=\beta} = \frac{ikl \cos \alpha}{1 + ikl \cos \alpha}$$

and

$$T_{\alpha=\beta} = \frac{1}{1 + ikl \cos \alpha}$$

where $k = k_1 = k_2$. For $\alpha = 0$ (normal incidence), these equations become, respectively,

$$R_{\alpha=0} = \frac{ikl}{1 + ikl}$$

and

$$T_{\alpha=0} = \frac{1}{1 + ikl}$$

which are the coefficients obtained by Lamb (ref. 7) for a plane sound wave incident normally on a slotted wall.

The energy relationship at the boundary of a moving fluid stream in terms of the intensities of the incident, reflected, and transmitted waves (provided the transmitted wave exists) is given by the equation

$$1 - |R|^2 = |T_p|^2 \frac{\sin 2\beta}{\sin 2\alpha} \tag{16}$$

This equation was derived by Ribner (ref. 5) under very general conditions and is applicable to the slotted boundary. In this connection, it should be reasserted that there is no increase in entropy at the wall, as the wall has no "resistive" effect but is a pure "inductance." It is not difficult to show that equations (10) and (11) are consistent with equation (16) if the notation is first simplified by the following substitutions:

$$A = \sin 2\alpha$$

$$B = \sin 2\beta$$

$$C = lk_R(\sin 2\beta \cot \alpha + \sin 2\alpha \cot \beta)$$

With this notation, using the expression for R given in equation (10) to solve for $1 - |R|^2$ gives

$$1 - |R|^2 = \frac{\left(A^2 + 2AB + B^2 + C^2\right)^2 - \left(A^2 - B^2 + C^2\right)^2 - 4B^2C^2}{\left[(A + B)^2 + C^2\right]^2}$$

$$= \frac{\left[\left(A^2 - B^2 + C^2\right) + 2B(A + B)\right]^2 - \left(A^2 - B^2 + C^2\right)^2 - 4B^2C^2}{\left[(A + B)^2 + C^2\right]^2}$$

$$= \frac{4B(A + B)\left(A^2 - B^2 + C^2\right) + 4B^2(A + B)^2 - 4B^2C^2}{\left[(A + B)^2 + C^2\right]^2}$$

$$= \frac{4AB\left[(A + B)^2 + C^2\right]}{\left[(A + B)^2 + C^2\right]^2}$$

$$= |T_p|^2 \frac{\sin 2\beta}{\sin 2\alpha}$$

LIMITATIONS OF THE THEORY

This analysis of an idealized physical situation is not likely to yield highly accurate quantitative results. An important difference between a real physical flow and the idealized flow is the mixing region that develops at the interface. This analysis should yield good qualitative results, however, when the sound waves are long compared with the thickness of the mixing region (ref. 5). The use of equation (6) to approximate the virtual mass of the boundary has been shown in reference 10 to give accurate results in predicting the resonant frequencies of slit resonators.

It should also be reasserted that the only intended purpose of this analysis relative to the problem of tunnel-wall interference on an oscillating airfoil in a slotted-throat wind tunnel is to discuss the boundary conditions, which should be the same for both the acoustics and aerodynamic problems. Perhaps the greatest mathematical difficulty in the wind-tunnel problem lies in the fact that the angle and amplitude of the incident waves vary along the boundary. The difficulties involved in correlating the results of a solution of the idealized mathematical problem with the actual flow would be increased by the fact that a wind-tunnel test section has limited rather than infinite length. This factor is believed to be of more significance in unsteady flow than in steady flow. Furthermore, the sound waves transmitted through the boundary would be reflected back to the boundary from the walls of the tank surrounding the test section.

The fact that there is, in general, a considerable temperature difference across the wind-tunnel wall does not complicate the problem. As long as the differences in density ρ and velocity of sound c on the two sides of the boundary are due solely to the temperature difference, the value of the expression Γ in equations (14) and (15) is unity, since the temperature ratio is directly proportional to the ratio of the squares of the sound velocities and is inversely proportional to the ratio of the densities.

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APPENDIX

DETAILS OF COMPUTATION OF REFLECTION AND TRANSMISSION COEFFICIENTS

The expressions given in equations (10), (11), and (13) are obtained from the boundary conditions as follows:

Substituting the relationships given in equations (4) and (5) into equations (1), (2), and (3) yields, respectively,

$$\varphi_i = \exp\left[ik_R(x - Ut)\right]\exp\left(ik_R y \cot \alpha\right) \tag{17}$$

$$\varphi_r = R \exp\left[ik_R(x - Ut)\right]\exp\left(-ik_R y \cot \alpha\right) \tag{18}$$

$$\varphi_t = T \exp\left[ik_R(x - Ut)\right]\exp\left(ik_R y \cot \beta\right) \tag{19}$$

Solving equation (4) for $\sin \alpha$ and $\sin \beta$ gives

$$\left.\begin{aligned} \sin \alpha &= \frac{c}{U - V_1} \\ \sin \beta &= \frac{c}{U - V_2} \end{aligned}\right\} \tag{20}$$

Then,

$$\left.\begin{aligned} \cot \alpha &= \left[\left(\frac{U - V_1}{c}\right)^2 - 1\right]^{1/2} \\ \cot \beta &= \left[\left(\frac{U - V_2}{c}\right)^2 - 1\right]^{1/2} \\ \cos \alpha &= \sin \alpha \cot \alpha \\ \cos \beta &= \sin \beta \cot \beta \end{aligned}\right\} \tag{21}$$

If the quantity $\exp\left[ik_R(x - Ut)\right]$ be denoted by φ_0, the incident pressure at the boundary can be written, from equations (1) and (8), as

$$p_i\Big]_{y=0} = -\rho\left(\frac{\partial\varphi_i}{\partial t}\right)_{y,x-V_1t}\Bigg]_{y=0} = ik_1\rho c\varphi_0$$

Similarly,

$$p_r\Big]_{y=0} = Rik_1\rho c\varphi_0$$

$$p_t\Big]_{y=0} = Tik_2\rho c\varphi_0$$

Equation (7) can now be written as

$$-i\rho c\varphi_0\left[k_1(1 + R) - k_2T\right] = \rho c l\varphi_0\left[k_1^2(1 - R)\cos\alpha + k_2^2T\cos\beta\right] \tag{22}$$

and equation (9) becomes

$$\frac{\sin\alpha}{c}\left(ik_1\cos\alpha\right)(1 - R)\varphi_0 = \frac{\sin\beta}{c}\left(ik_2\cos\beta\right)T\varphi_0 \tag{23}$$

Reducing equations (22) and (23) and solving for $\dfrac{1 + R}{1 - R}$ yield

$$\frac{1 + R}{1 - R} = \frac{k_2\cos\alpha}{k_1\cos\beta} + il\left(k_1\cos\alpha + \frac{k_2^2}{k_1}\cos\alpha\right) \tag{24}$$

Substituting the relationships given in equation (5) into equation (24) yields

$$\frac{1 + R}{1 - R} = \frac{\sin 2\alpha}{\sin 2\beta} + \frac{ilk_R(\sin 2\beta\cot\alpha + \sin 2\alpha\cot\beta)}{\sin 2\beta} \tag{25}$$

Solving equation (25) for R gives equation (10). Then, T as given in equation (11) may be found by substituting the relationship given in equation (10) into equation (23).

REFERENCES

1. Runyan, Harry L., and Watkins, Charles E.: Considerations on the Effect of Wind-Tunnel Walls on Oscillating Air Forces for Two-Dimensional Subsonic Compressible Flow. NACA Rep. 1150, 1953. (Supersedes NACA TN 2552.)

2. Drake, D. G.: The Motion of an Oscillating Aerofoil in a Compressible Free Jet. Jour. R.A.S., vol. 60, no. 549, Sept. 1956, pp. 621-623.

3. Drake, D. G.: The Oscillating Two-Dimensional Aerofoil Between Porous Walls. Aero. Quarterly, vol. VIII, pt. 3, Aug. 1957, pp. 226-239.

4. Miles, John W.: On the Reflection of Sound at an Interface of Relative Motion. Jour. Acous. Soc. of America, vol. 29, no. 2, Feb. 1957, pp. 226-228.

5. Ribner, Herbert S.: Reflection, Transmission, and Amplification of Sound by a Moving Medium. Jour. Acous. Soc. of America, vol. 29, no. 4, Apr. 1957, pp. 435-441.

6. Rayleigh, (Lord): The Theory of Sound. First Am. ed., vol. II, Dover Publications, 1945, pp. 80, 133.

7. Lamb, Horace: Hydrodynamics. Sixth ed., Dover Publications, 1945, pp. 533-537.

8. Davis, Don D., Jr., and Moore, Dewey: Analytical Study of Blockage- and Lift-Interference Corrections for Slotted Tunnels Obtained by the Substitution of an Equivalent Homogeneous Boundary for the Discrete Slots. NACA RM L53E07b, 1953.

9. Morse, Philip M., and Bolt, Richard H.: Sound Waves in Rooms. Rev. Modern Phys., vol. 16, no. 2, Apr. 1944, pp. 69-150.

10. Smits, J. M. A., and Kosten, C. W.: Sound Absorption by Slit Resonators. Acustica, vol. 1, no. 3, 1951, pp. 114-122.

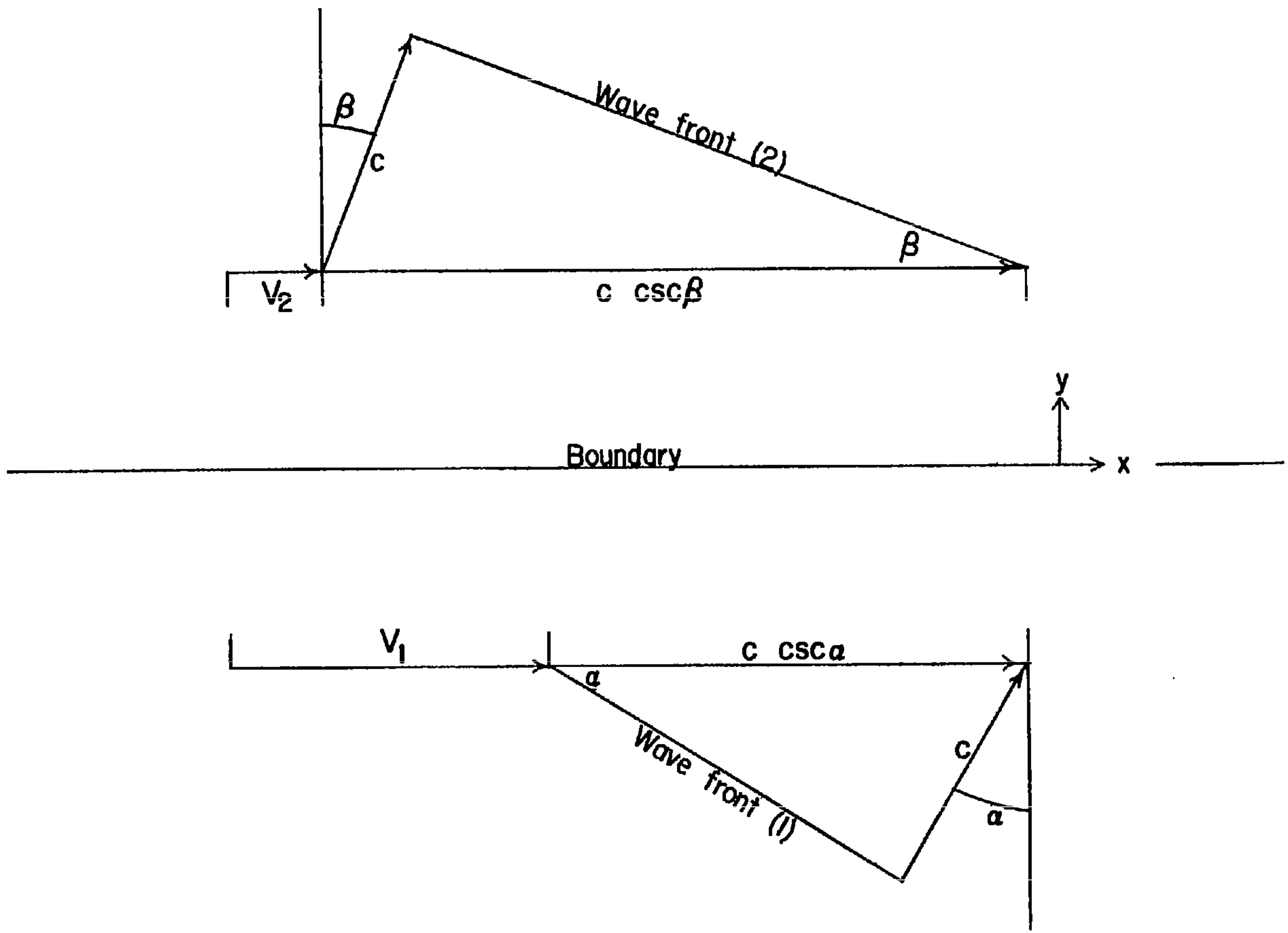

Figure 1.- Side view of boundary with velocity vectors and coordinate system fixed in space.

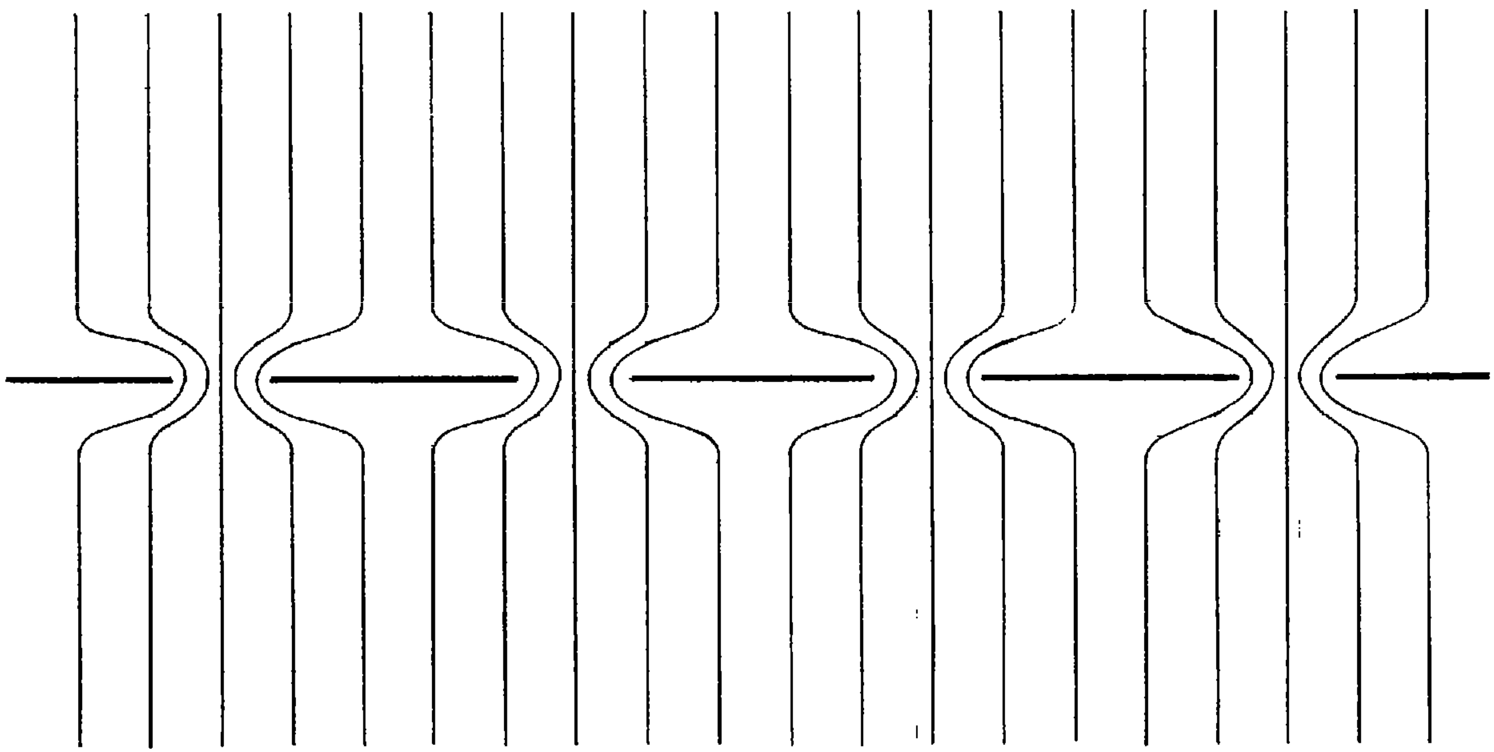

Figure 2.- Section of wall showing streamlines of normal flow through slots.